An Optimization Technique for Lumped-Distributed Two Ports

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Abstract —**A frequency domain direct efficient analysis and an optimization technique of a large class of lumped-distributed networks containing active elements are presented. Sensitivity and Hessian matrix calculations are performed using truncated Taylor series expansion of two-port parameters of subnetworks. An interactive computer program was developed to demonstrate the application of the method. Examples of network optimizations are included to illustrate the powerfulness of the technique.**

I. Introduction

Most MICROWAVE integrated circuit designs require a significant amount of computer aided analysis and optimization. In the past decade, techniques for network sensitivity and optimization have received considerable attention. These techniques generally belonged to two broad categories: the indirect approach which utilizes the concepts of adjoint networks [1], [2], and other direct approaches that utilize the inverse of a nodal admittance matrix [3], [4].

This paper presents a new direct efficient approach to the analysis and optimization of a large class of lumped-distributed networks containing active elements. The class of networks under consideration includes microwave integrated circuits and other networks that possess sparse nodal admittance matrices. Although it is well known that such a class of networks is more efficiently analyzed as an interconnection of subnetwork m-ports [5], [6], little has been done to extend this direct analysis approach to the computation of network sensitivities [7]. The development of such an approach for the frequency domain analysis and optimization of lumped-distributed two-port networks using the Hessian matrix is the subject of this paper. In this approach, a two-port network is analyzed as the interconnection of several subnetworks expanded in a truncated nth-order Taylor series of their two-port parameters. Direct analysis of the two-port network is performed by converting each subnetwork to an appropriate domain for interconnection, then linearly combining their Taylor series terms. The number of terms used in the expansion determines the highest order of network sensitivity available for optimization.

This direct approach to network sensitivity analysis is more efficient than conventional adjoint and inverse nodal admittance matrix approaches because it does not require complex nodal admittance matrix inversion, nor repeated analyses. A comparison of these three basic approaches to network sensitivity analysis shows that computations for second-order direct analysis increase linearly with the number of interconnected subnetworks and as the square of the number of variable network elements. For the adjoint and inverse nodal admittance analysis, the required computations increase with the cube of the number of nodes.

To demonstrate the powerfulness of this new approach, an interactive computer program was developed for the analysis and optimization of lumped-distributed two ports containing active elements in the frequency domain. The program accepts any set of eighteen lumped-distributed two-port subnetwork types including: uniform transmission lines, RLC subnetworks, controlled sources, ideal transformers, mutually coupled coils, gyrators, and negative converters. Permissible interconnections include: series, parallel, cascade, and hybrid interconnections in the impedance, admittance, hybrid, transmission, and scattering parameter domains.

Section II briefly describes the methods used in the sensitivity computations of interconnected networks. Section III presents the analysis and optimization technique, including error function definitions, various algorithms used for optimization, and the strategy of the optimization program. Examples of network optimizations are given in Section IV, including an octave bandwidth FET amplifier and a lumped circuit approximating a uniformly distributed RC line, which demonstrate the powerfulness of this new approach.

II. Sensitivity Analysis of Interconnected Two-Port Networks

The class of networks that can be analyzed and optimized using the technique described in this paper is linear two ports which can be decomposed as an interconnection of two-port subnetworks. The interconnections between any two subnetworks can be one of the commonly used interconnections [8], namely series, parallel, hybrid, and cascade. However, for simplicity and to keep the length of this paper within a reasonable limit, the process is illustrated here only for the case of cascaded subnetworks shown in Fig. 1. The optimization procedure of the network involves the minimization of a properly chosen error function. The error function is expressed as a truncated second-order Taylor series expansion. This expansion involves only subnetwork two-port parameters and their

Manuscript received March 19, 1982; revised June 28, 1982.

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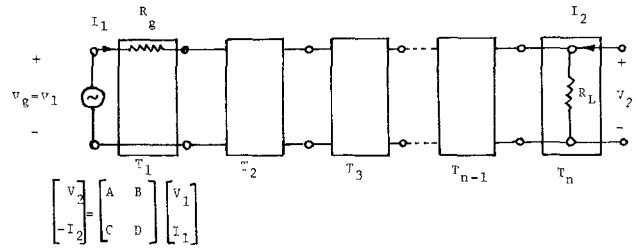


Fig. 1. Cascaded two-port.

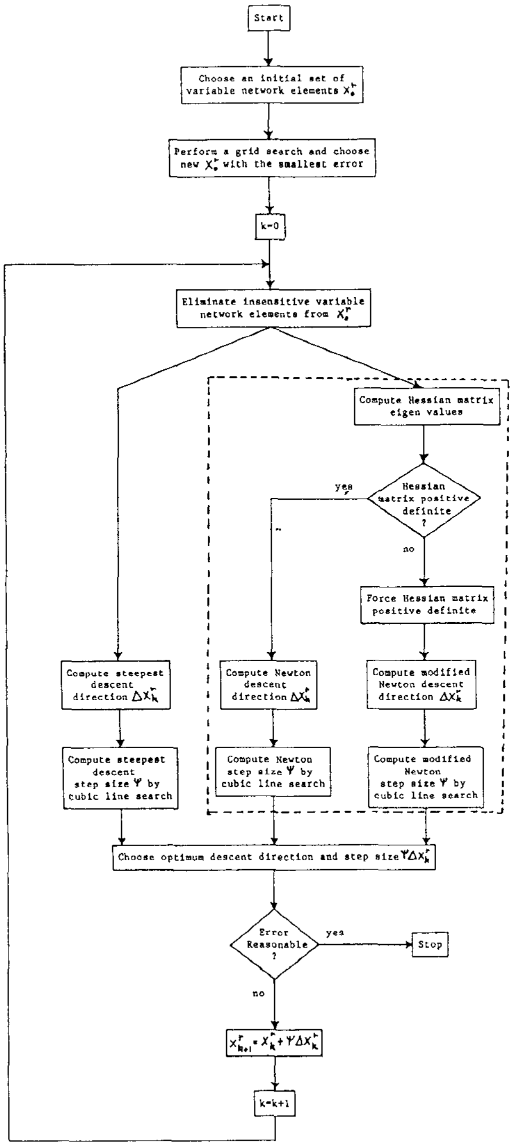


Fig. 2. Optimization strategy.

partial derivatives. For a cascaded network the required partial derivatives are calculated from the transmission matrix of the subnetworks that contain the variable element. Thus, for the cascaded network in Fig. 1 which consists of n subnetworks, the overall transmission matrix is expressed as

$$T(\boldsymbol{x}) = T_1 T_2 \cdots T_{k-1} T_k T_{k+1} \cdots T_n \tag{1}$$

where T_k is the transmission matrix of the kth subnetwork of the cascade, $k = 1, 2, \cdots, n$, and $\boldsymbol{x}$ is a vector of the natural logarithm of the r variable network elements. If x_i belongs only to subnetwork l, x_j only to subnetwork k, then the partial derivatives of $T(\boldsymbol{x})$ with respect to x_i and x_j are given by

$$\frac{\partial^p T}{\partial x_i^p} = T_1 T_2 \cdots T_{l-1} \frac{\partial^p T_l}{\partial x_i^p} T_{l+1} \cdots T_n, \qquad p = 1,2 \tag{2a}$$

$$\frac{\partial^p T}{\partial x_j^p} = T_1 T_2 \cdots T_{k-1} \frac{\partial^p T_k}{\partial x_j^p} T_{k+1} \cdots T_n, \qquad p = 1,2 \tag{2b}$$

$$\frac{\partial^2 T}{\partial x_i \partial x_j} = T_1 T_2 \cdots T_{l-1} \frac{\partial T_l}{\partial x_i} T_{l+1} \cdots \cdot T_{k-1} \frac{\partial T_k}{\partial x_j} T_{k+1} \cdots T_n, \qquad i \neq j. \tag{2c}$$

Table I shows some of the more common circuit elements with their transmission matrices and their first- and second-order partial derivatives. This process is used to compute very effectively the required derivatives of the error function in the various algorithms used in the minimization. The same technique can be used to analyze any interconnected two-port network, not only the cascaded. Details of this process are given in [8] using tensor notation to describe the interconnection of subnetworks, even those requiring two-port parameter domain conversion.

III. Optimization of Two-Port Networks

Optimization of the network starts by choosing a suitable error function to measure the difference between desired and actual two-port frequency responses of the network. This error function should have a zero value when the desired and actual frequency responses of the network are equal, and a value approaching positive infinity when they are unequal. A weighted least pth error function with even p is one such function which is used in a demonstration program to represent a reasonable measure of the error.

Network scaling is achieved in two ways. First, the network is analyzed with elements and frequencies centered about unity, and second, the network is optimized in a logarithmic vector space of variable network elements and network frequency responses. This second method of scaling smoothes the error function for better computational stability and more dynamic range.

As shown in Fig. 2, six basic algorithms are used for network optimization. They are: an initial nonsequential grid search, an algorithm for eliminating insensitive variable network elements, the method of steepest descent, Newton's method, a modified Newton method for handling nonpositive definite Hessian matrices, and a cubic line search. Only one of these algorithms, the method of steepest descent, is absolutely required for global convergence of the combined algorithm. All other algorithms are optional and are shown within the dotted borders in Fig. 2.

With the exception of variable network element elimination, all optimization algorithms attempt to minimize the

TABLE I
CIRCUIT ELEMENTS, TRANSMISSION MATRICES, AND SENSITIVITIES

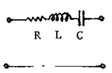
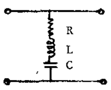
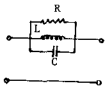
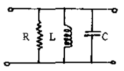
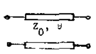
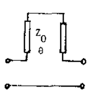
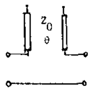
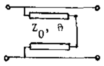
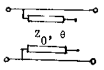
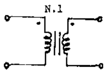
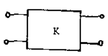
CIRCUIT	X_i	T	$\frac{\partial T}{\partial X_i}$	$\frac{\partial^2 T}{\partial X_i^2}$	X_j	$\frac{\partial^2 T}{\partial X_i \partial X_j}$
Series-Series R,L,C 	$\ell n\ R$	$\begin{bmatrix}1 & Z\\0 & 1\end{bmatrix}$ $Z=R+j(\omega L-\frac{1}{\omega C})$	$\begin{bmatrix}0 & R\\0 & 0\end{bmatrix}$	$\begin{bmatrix}0 & R\\0 & 0\end{bmatrix}$	$\ell n\ L$ or $\ell n\ C$	$\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$
	$\ell n\ L$		$\begin{bmatrix}0 & j\omega L\\0 & 0\end{bmatrix}$	$\begin{bmatrix}0 & j\omega L\\0 & 0\end{bmatrix}$	$\ell n\ C$	$\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$
	$\ell n\ C$		$\begin{bmatrix}0 & \frac{-1}{j\omega C}\\0 & 0\end{bmatrix}$	$\begin{bmatrix}0 & \frac{1}{j\omega C}\\0 & 0\end{bmatrix}$		
Series-Parallel R,L,C 	$\ell n\ R$	$\begin{bmatrix}1 & 0\\Y & 1\end{bmatrix}$ $Y=\frac{1}{R+j\omega L+\frac{1}{j\omega C}}$	$\begin{bmatrix}0 & 0\\-RY^3 & 0\end{bmatrix}$	$\begin{bmatrix}0 & 0\\2R^3Y^3-RY^2 & 0\end{bmatrix}$	$\ell n\ L$	$\begin{bmatrix}0 & 0\\2j\omega LRY^3 & 0\end{bmatrix}$
					$\ell n\ C$	$\begin{bmatrix}0 & 0\\\frac{j2RY^3}{\omega C} & 0\end{bmatrix}$
	$\ell n\ L$		$\begin{bmatrix}0 & 0\\-j\omega LY^2 & 0\end{bmatrix}$	$\begin{bmatrix}0 & 0\\j\omega LY^2(2j\omega LY-1) & 0\end{bmatrix}$	$\ell n\ C$	$\begin{bmatrix}0 & 0\\\frac{-2LY^3}{C} & 0\end{bmatrix}$
	$\ell n\ C$		$\begin{bmatrix}0 & 0\\\frac{2jRY^3}{\omega C} & 0\end{bmatrix}$	$\begin{bmatrix}0 & 0\\\frac{Y^2}{j\omega C}(\frac{2Y}{j\omega C}-1) & 0\end{bmatrix}$		
Parallel-Series R,L,C 	$\ell n\ R$	$\begin{bmatrix}1 & Z\\0 & 1\end{bmatrix}$ $\frac{1}{Z}=\frac{1}{R}+\frac{1}{j\omega L}+j\omega C$	$\begin{bmatrix}0 & \frac{Z^2}{R}\\0 & 0\end{bmatrix}$	$\begin{bmatrix}0 & \frac{Z^2}{R}(2\frac{Z}{R}-1)\\0 & 0\end{bmatrix}$	$\ell n\ L$	$\begin{bmatrix}0 & \frac{2Z^3}{j\omega LR}\\0 & 0\end{bmatrix}$
	$\ell n\ L$		$\begin{bmatrix}0 & \frac{Z^2}{j\omega L}\\0 & 0\end{bmatrix}$	$\begin{bmatrix}0 & \frac{Z^2}{j\omega L}(\frac{2Z}{j\omega L}-1)\\0 & 0\end{bmatrix}$	$\ell n\ C$	$\begin{bmatrix}0 & \frac{-2j\omega CZ^3}{R}\\0 & 0\end{bmatrix}$
	$\ell n\ C$		$\begin{bmatrix}0 & -j\omega CZ^2\\0 & 0\end{bmatrix}$	$\begin{bmatrix}0 & j\omega CZ^2(2j\omega CZ-1)\\0 & 0\end{bmatrix}$	$\ell n\ C$	$\begin{bmatrix}0 & \frac{-2CZ^3}{L}\\0 & 0\end{bmatrix}$
Parallel-Parallel R,L,C 	$\ell n\ R$	$\begin{bmatrix}1 & 0\\Y & 1\end{bmatrix}$ $Y=\frac{1}{R}+\frac{1}{j\omega L}+j\omega C$	$\begin{bmatrix}0 & 0\\-\frac{1}{R} & 0\end{bmatrix}$	$\begin{bmatrix}0 & 0\\\frac{1}{R} & 0\end{bmatrix}$	$\ell n\ L$ or $\ell n\ C$	$\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$
	$\ell n\ L$		$\begin{bmatrix}0 & 0\\\frac{1}{\omega L} & 0\end{bmatrix}$	$\begin{bmatrix}0 & 0\\-\frac{1}{j\omega L} & 0\end{bmatrix}$	$\ell n\ C$	$\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$
	$\ell n\ C$		$\begin{bmatrix}0 & 0\\j\omega C & 0\end{bmatrix}$	$\begin{bmatrix}0 & 0\\j\omega C & 0\end{bmatrix}$		
Uniform, Lossless Transmission Line 	$\ell n\ Z_0$	$\begin{bmatrix}\cos\theta & jZ_0\sin\theta\\jY_0\sin\theta & \cos\theta\end{bmatrix}$ $Y_0=\frac{1}{Z_0}$	$\begin{bmatrix}0 & jZ_0\sin\theta\\-jY_0\sin\theta & 0\end{bmatrix}$	$\begin{bmatrix}0 & jZ_0\sin\theta\\jY_0\sin\theta & 0\end{bmatrix}$	$\ell n\ \theta$	$\begin{bmatrix}0 & jZ_0\theta\cos\theta\\-jY_0\theta\cos\theta & 0\end{bmatrix}$
	$\ell n\ \theta$		$\begin{bmatrix}-\theta\sin\theta & jZ_0\theta\cos\theta\\jY_0\theta\cos\theta & -\theta\sin\theta\end{bmatrix}$	$\begin{bmatrix}-a & jZ_0b\\jY_0b & -a\end{bmatrix}$ $a=\theta^2\cos\theta+\theta\sin\theta$ $b=\theta\cos\theta-\theta^2\sin\theta$		
Series Shorted Stub 	$\ell n\ Z_0$	$\begin{bmatrix}1 & Z\\0 & 1\end{bmatrix}$ $Z=jZ_0\tan\theta$	$\begin{bmatrix}0 & Z\\0 & 0\end{bmatrix}$	$\begin{bmatrix}0 & Z\\0 & 0\end{bmatrix}$	$\ell n\ \theta$	$\begin{bmatrix}0 & \frac{jZ_0\theta}{\cos^2\theta}\\0 & 0\end{bmatrix}$
	$\ell n\ \theta$		$\begin{bmatrix}0 & \frac{jZ_0\theta}{\cos^2\theta}\\0 & 0\end{bmatrix}$	$\begin{bmatrix}0 & jZ_0c\\0 & 0\end{bmatrix}$ $c=\frac{\theta\cos\theta+2\theta^2\sin\theta}{\cos^3\theta}$		

TABLE I (*Continued*)

CIRCUIT	X_i	T	$\frac{\partial T}{\partial X_i}$	$\frac{\partial^2 T}{\partial X_i^2}$	X_j	$\frac{\partial^2 T}{\partial X_i \partial X_j}$
Series Opened Stub 	$\ell n\ Z_0$	$\begin{bmatrix}1 & Z\\0 & 1\end{bmatrix}$ $Z=-jZ_0\cot\theta$	$\begin{bmatrix}0 & Z\\0 & 0\end{bmatrix}$	$\begin{bmatrix}0 & Z\\0 & 0\end{bmatrix}$	$\ell n\ \theta$	$\begin{bmatrix}0 & \frac{-jZ_0\theta}{\sin^2\theta}\\0 & 0\end{bmatrix}$
	$\ell n\ \theta$		$\begin{bmatrix}0 & \frac{-jZ_0\theta}{\sin^2\theta}\\0 & 0\end{bmatrix}$	$\begin{bmatrix}0 & jZ_0d\\0 & 0\end{bmatrix}$ $d=\frac{\theta\sin\theta-2\theta^2\cos\theta}{\sin^3\theta}$		
Shunt Shorted Stub 	$\ell n\ Z_0$	$\begin{bmatrix}1 & 0\\Y & 1\end{bmatrix}$ $Y=\frac{-j\cot\theta}{Z_0}$	$\begin{bmatrix}0 & 0\\-Y & 0\end{bmatrix}$	$\begin{bmatrix}0 & 0\\Y & 0\end{bmatrix}$	$\ell n\ \theta$	$\begin{bmatrix}0 & 0\\\frac{j\theta}{Z_0\sin^2\theta} & 0\end{bmatrix}$
	$\ell n\ \theta$		$\begin{bmatrix}0 & 0\\\frac{-j\theta}{Z_0\sin^2\theta} & 0\end{bmatrix}$	$\begin{bmatrix}0 & 0\\\frac{jd}{Z_0} & 0\end{bmatrix}$ $d=\frac{\theta\sin\theta-2\theta^2\cos\theta}{\sin^3\theta}$		
Shunt Opened Stub 	$\ell n\ Z_0$	$\begin{bmatrix}1 & 0\\Y & 1\end{bmatrix}$ $Y=\frac{j\tan\theta}{Z_0}$	$\begin{bmatrix}0 & 0\\-Y & 0\end{bmatrix}$	$\begin{bmatrix}0 & 0\\Y & 0\end{bmatrix}$	$\ell n\ \theta$	$\begin{bmatrix}0 & 0\\\frac{-j\theta}{Z_0\cos^2\theta} & 0\end{bmatrix}$
	$\ell n\ \theta$		$\begin{bmatrix}0 & 0\\\frac{j\theta}{Z_0\cos^2\theta} & 0\end{bmatrix}$	$\begin{bmatrix}0 & 0\\\frac{jc}{Z_0} & 0\end{bmatrix}$ $c=\frac{\theta\cos\theta+2\theta^2\sin\theta}{\cos^3\theta}$		
Ideal Transformer 	$\ell n\ N$	$\begin{bmatrix}N & 0\\0 & \frac{1}{N}\end{bmatrix}$	$\begin{bmatrix}N & 0\\0 & \frac{-1}{N}\end{bmatrix}$	$\begin{bmatrix}N & 0\\0 & \frac{1}{N}\end{bmatrix}$		
K-Inverter 	$\ell n\ K$	$\begin{bmatrix}0 & jK\\\frac{-1}{K} & 0\end{bmatrix}$	$\begin{bmatrix}0 & jK\\\frac{1}{K} & 0\end{bmatrix}$	$\begin{bmatrix}0 & jK\\\frac{-1}{K} & 0\end{bmatrix}$		

local error function by simultaneously varying all variable network elements. The way these network elements are varied is different for each minimization algorithm based on the sensitivity information required by the algorithm. One of these algorithms, the grid search, requires no sensitivity information. This algorithm executes faster than the other algorithms, but gives the lowest rate of convergence.

The method of steepest descent and the cubic line search require first-order sensitivity information, and execute slower than the grid search but with a much better rate of convergence. The Newton method algorithms both require second-order sensitivity information and execute the slowest but with the best rate of convergence, approaching second-order. Therefore, the selection of which algorithms to use for a particular network optimization becomes almost as important as the choice of initial variable network elements. This selection should be based on the accuracy of initial variable network elements, the computational costs for analysis and optimization, and past experience with similar networks.

A. Error Function Definition

In the circuit of Fig. 1, the source and load resistances R_g and R_L are taken to be part of the cascade. If the overall transmission matrix of the network is A, B, C, D, as defined in the figure, then the gain error function ϵ_g may be defined as

$$\epsilon_g = \sum_{\text{freq}} \left(AA^* - \frac{4R_g}{R_L G_0} \right)^p \tag{3a}$$

where G_0 is a desired power gain. Similarly error functions for the output and input reflection coefficients ϵ_{Γ_1} and ϵ_{Γ_1} can be defined, respectively, by

$$\epsilon_{\Gamma_2} = \sum_{\text{freq}} \left[\left(\frac{A}{B} - \frac{2}{R_L} \right) \left(\frac{A^*}{B^*} - \frac{2}{R_L} \right) \right]^p \tag{3b}$$

$$\epsilon_{\Gamma_1} = \sum_{\text{freq}} \left[\left(\frac{D}{C} - \frac{2}{R_g} \right) \left(\frac{D^*}{C^*} - \frac{2}{R_g} \right) \right]^p \tag{3c}$$

where p is a positive integer. An overall weighted error function can be defined as

$$\epsilon = w_g \epsilon_g + w_1 \epsilon_{\Gamma_1} + w_2 \epsilon_{\Gamma_2} \tag{4}$$

where w_g, w_1, and w_2 are suitable nonnegative weighting coefficients. It is clear that all the partial derivatives of ϵ, with respect to the natural logarithm of any of the network elements, involve only the partial derivatives of the transmission matrix elements A, B, C, and D, all of which are readily computable as explained in Section II.

IV. Examples of Network Optimization

Two examples of networks are chosen to demonstrate the powerfulness of the present optimization technique. The first network is a wide-band single-stage FET amplifier consisting of input and output matching networks, each of which has three stubs and a single cascaded line as shown in Fig. 3 [9]. The FET is defined in Table II by scattering parameters at seven frequencies from 6 to 12 GHz, inclusive. All the transmission lines and stubs have fixed characteristic impedances and variable lengths. The network was optimized for a constant 8-dB power gain over the frequency band of 6 to 12 GHz. The error function used in the optimization is as given in (3) and (4), with the values of $p = 2$, $w_1 = w_2 = 0$, and $w_g = 1$. This network was optimized in 5 iterations using the present technique. The solution obtained by the present technique indicated that the last stub can be removed. Initial and final values of the line lengths and amplifier response are given in Table III and Fig. 4, respectively.

The second example is a lumped RC ladder network, shown in Fig. 5, which was previously analyzed and optimized by Wing and Behar [4] using inverse nodal admittance analysis to compute first- and second-order network element sensitivities. The network contains seven variable network elements, four capacitances, and three resistances, and is optimized over a logarithmic range of ten frequencies between 0.1 and 10 radians. The desired input impedance over this range was that of a distributed RC line.

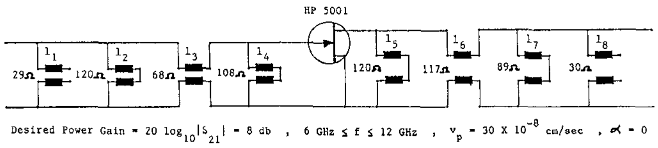


Fig. 3. FET amplifier circuit.

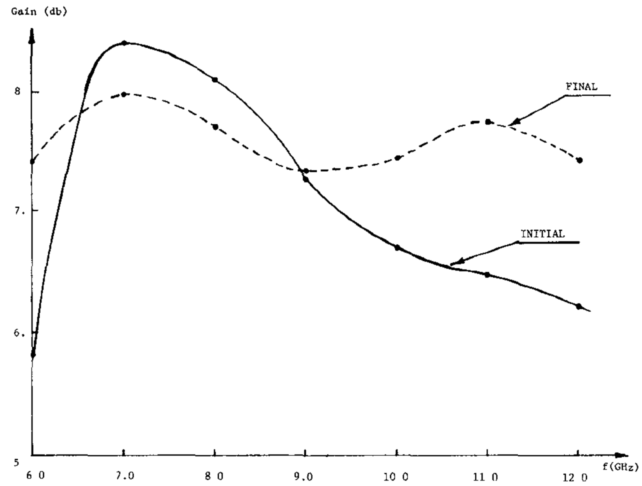


Fig. 4. Initial and final response of FET amplifier example.

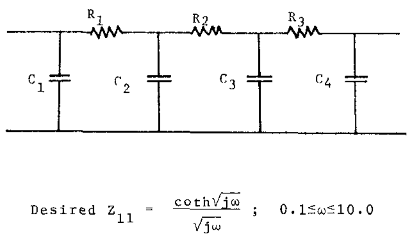


Fig. 5. Lumped model of a distributed RC line.

TABLE II
FET Scattering Parameters

f (GHz)	S_{11}	S_{12}	S_{21}	S_{22}
6	0.73/-96°	0.051/69.8°	1.97/99.3°	0.64/-26°
7	0.69/-110°	0.059/70.3°	1.84/88.2°	0.63/-30°
8	0.67/-124°	0.066/73.2°	1.71/78.4°	0.62/-35°
9	0.65/-136°	0.073/77.2°	1.56/69.8°	0.61/-40°
10	0.62/-147°	0.081/82.7°	1.45/61.7°	0.61/-45°
11	0.61/-155°	0.093/87.5°	1.38/53.7°	0.60/-50°
12	0.62/-160°	0.108/90.7°	1.33/44.6°	0.60/-58°

TABLE III
Variable Line Lengths for FET Amplifier Example

	Initial	Final
l_1	0.2986 cm	0.3247 cm
l_2	0.2986	0.4074
l_3	0.2986	0.2113
l_4	0.2986	0.2740
l_6	0.2986	0.3727
l_7	0.2986	1.012
l_8	0.2986	0.001198

TABLE IV
VARIABLE NETWORK ELEMENTS FOR LUMPED *RC* LINE EXAMPLE

	Initial	Final
C_1	0.1250	0.04319
R_1	0.2500	0.1258
C_2	0.2500	0.1973
R_2	0.5000	0.2512
C_2	0.5000	0.2866
R_3	1.000	0.3276
C_4	1.000	0.4729

TABLE V
INPUT IMPEDANCE FOR LUMPED *RC* LINE EXAMPLE

Log w	Initial Z_{11}	Final and Desired Z_{11}
-1.0	5.4264 $\angle$ -81.301°	10.008 $\angle$ -88.091°
-0.8	3.5103 $\angle$ -76.518°	6.3219 $\angle$ -86.978°
-0.6	2.3447 $\angle$ -69.718°	4.006 $\angle$ -85.222°
-0.4	1.6621 $\angle$ -61.265°	2.5426 $\angle$ -82.475°
-0.2	1.2776 $\angle$ -53.083°	1.6331 $\angle$ -78.252°
0.0	1.0487 $\angle$ -47.964°	1.0744 $\angle$ -72.042°
0.2	0.87116 $\angle$ -47.033°	0.74188 $\angle$ -63.748°
0.4	0.70003 $\angle$ -48.554°	0.55326 $\angle$ -54.564°
0.6	0.54582 $\angle$ -50.071°	0.44761 $\angle$ -47.151°
0.8	0.42426 $\angle$ -51.302°	0.37773 $\angle$ -43.688°
1.0	0.32867 $\angle$ -53.267°	0.31451 $\angle$ -43.729°

Direct optimization of the *RC* ladder network required 20 iterations, including the initial grid search. This compares favorably to Wing and Behar's optimization of the same network which took 44 iterations. Newton's method gave the lowest step error in 12 of these iterations, and the method of steepest descent in only 7 of these iterations. Variable network elements were eliminated in only one iteration, indicating that the optimized input impedance was sensitive to all variable network elements most of the time. Initial and final variable network elements for this network optimization are shown in Table IV with initial and final input impedances for all frequencies in Table V. The optimized final values are identical in both magnitude and phase to the desired input impedances at all frequencies shown. The error function used in the optimization is that of (3) and (4) with $p = 2$, $w_1 = 1$, $w_2 = w_g = 0$.

V. CONCLUSION

A technique is presented for the optimization of a large class of lumped-distributed microwave networks containing active elements. This technique computes the network sensitivities and the Hessian matrix directly, using an efficient method which does not require repeated analysis nor the inversion of a nodal admittance matrix. The optimization strategy used in a demonstration program is described and examples of a wide-band single-stage FET amplifier and a lumped *RC* ladder were used as illustrations of the powerfulness of the technique.

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